

Absorption cross section in Lifshitz black hole

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Abstract

We derive the absorption cross section of a minimally coupled scalar in the Lifshitz black hole obtained from the new massive gravity. The absorption cross section reduces to the horizon area in the low energy and massless limits of scalar propagation, indicating that the Lifshitz black hole also satisfies the universality of low-energy absorption cross section for black holes.

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1 Introduction

The Lifshitz-type black holes [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17] have received considerable attentions since these may provide a model of generalizing AdS/CFT correspondence to non-relativistic condensed matter physics [the Lif/CFT correspondence]. Although their asymptotic spacetimes are known to be Lifshitz, the whole properties of these black holes are not yet explored completely.

Especially, we wish to focus on the ($z = 3$) Lifshitz black hole [18] derived from the new massive gravity [19] because it may be considered as a toy model for Lifshitz black holes.

If a black hole is found, a thermodynamic study is important to understand the black hole because heat capacity and free energy determine the thermodynamic stability of the black hole. Recently, there was a progress on computation of mass and related thermodynamic quantities by using the ADT method [20, 21] and Euclidean action approach in three dimensions [22]. There was a discrepancy in mass between $\mathcal{M} = \frac{7r_+^4}{8G\ell^4}$ obtained from the ADT method [20] and $\mathcal{M} = \frac{r_+^4}{4G\ell^4}$ from other approaches [22, 23, 24]. However, it turned out that the Lifshitz black hole is thermodynamically stable since its heat capacity is positive and free energy is negative. The possibility of phase transition between Lifshitz black hole and thermal Lifshitz has been discussed by introducing on-shell and off-shell free energies [25].

On the other hand, quasinormal modes of a black hole contain important information on the black hole. Its complex quasinormal frequency is given by $\omega = \omega_R - i\omega_I$ whose real part represents the oscillation and imaginary part denotes the rate at which this oscillation is damped, because of the very nature of black hole horizon. In addition, the condition of $\omega_I > 0$ is consistent with the stability condition of the black hole. Quasinormal frequencies (QNFs) were obtained from a perturbed scalar propagation by imposing the boundary condition: ingoing mode near horizon and Dirichlet boundary condition at infinity. Importantly, QNFs of Lifshitz black hole are purely imaginary [26, 27], showing that such perturbation has no oscillation stage. This feature may be related to the solitonic nature on the boundary field theory which implies that its equilibrium is comparatively stable and thus, it is difficult to take the theory out of equilibrium [28]. Consequently, this indicates that *the Lifshitz black hole is stable against an external perturbation, which is closely related to its thermodynamic stability.*

At this stage, one has to ask how the Lifshitz black hole is different from a three-dimensional known black hole of the non-rotating BTZ black hole [29, 30]. One difference is that QNFs of Lifshitz black hole are purely imaginary, whereas those of BTZ black hole are complex. Their thermodynamic property is the nearly same to each other: positive heat capacity and negative free energy [31] even though they have different asymptotes. One

remaining thing to explore is to compute the absorption cross section (=greybody factor) because it provides a litmus to test whether or not the Lifshitz black hole possesses the universal property of black holes [32].

In this work, we obtain the absorption cross section by investigating a minimally coupled scalar propagating in the Lifshitz black hole background. Importantly, we will show that the absorption cross section reduces to the horizon area in the low energy and massless limits of s -wave propagation, indicating that the Lifshitz black hole also satisfies the universality of low energy absorption cross section for any black holes.

2 Lifshitz black hole from new massive gravity

We start with the new massive gravity [19] composed of the Einstein-Hilbert action with a cosmological constant Λ and higher-order curvature terms given by

$$S_{\text{NMG}} = -[S_{\text{EH}} + S_{\text{HC}}], \quad (1)$$

$$S_{\text{EH}} = \frac{1}{16\pi G} \int d^3x \sqrt{-g} (R - 2\Lambda), \quad (2)$$

$$S_{\text{HC}} = -\frac{1}{16\pi G \tilde{m}^2} \int d^3x \sqrt{-g} \left(R_{\mu\nu} R^{\mu\nu} - \frac{3}{8} R^2 \right), \quad (3)$$

where G is a three-dimensional Newton constant and \tilde{m}^2 a parameter with mass dimension 2. We would like to mention that to avoid negative mass and entropy, it is necessary to take “ $-$ ” sign in the front of $[S_{\text{EH}} + S_{\text{HC}}]$. The field equation is given by

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} - \frac{1}{2\tilde{m}^2} K_{\mu\nu} = 0, \quad (4)$$

where

$$\begin{aligned} K_{\mu\nu} &= 2\Box R_{\mu\nu} - \frac{1}{2} \nabla_\mu \nabla_\nu R - \frac{1}{2} \Box R g_{\mu\nu} \\ &+ 4R_{\mu\nu\rho\sigma} R^{\rho\sigma} - \frac{3}{2} R R_{\mu\nu} - R_{\rho\sigma} R^{\rho\sigma} g_{\mu\nu} + \frac{3}{8} R^2 g_{\mu\nu}. \end{aligned} \quad (5)$$

In order to obtain the $z = 3$ Lifshitz black hole solution, we have to choose $\tilde{m}^2 = -\frac{1}{2\ell^2}$ and $\Lambda = -\frac{13}{2\ell^2}$ with ℓ the curvature radius of Lifshitz spacetimes. Explicitly, we find the $z = 3$ Lifshitz black hole solution [18] as

$$ds_{\text{Lif}}^2 = g_{\mu\nu} dx^\mu dx^\nu = - \left(\frac{r^2}{\ell^2} \right)^z \left(1 - \frac{M\ell^2}{r^2} \right) dt^2 + \frac{dr^2}{\left(\frac{r^2}{\ell^2} - M \right)} + r^2 d\phi^2, \quad (6)$$

where M is an integration constant related to the mass of black hole. The event horizon is located at $r = r_+ = \ell\sqrt{M}$. The $z = 1$ case corresponds to the BTZ black hole. The line element (6) is invariant under the anisotropic scaling of

$$t \rightarrow \lambda^z t, \quad \phi \rightarrow \lambda \phi, \quad r \rightarrow \frac{r}{\lambda} \quad (7)$$

with $M \rightarrow M/\lambda^2$. For $z = 1$ BTZ black hole, the ADM mass is determined to be $M = \frac{r_+^2}{\ell^2}$, while for $z = 3$ Lifshitz black hole, the ADM mass is proportional to M^2 . Its z -dependent curvature is given by

$$R_z = \frac{2}{\ell^2 r^2} \left[\ell^2 M - r^2 - (2\ell^2 M + r^2)z + (\ell^2 M - r^2)z^2 \right], \quad (8)$$

which yields

$$R_{z=1} = -\frac{6}{\ell^2}, \quad R_{z=3} = -\frac{26}{\ell^2} + \frac{8M}{r^2}. \quad (9)$$

3 Scalar propagation in Lifshitz spacetimes

In order to find the absorbtion cross section, we first consider a minimally coupled scalar described by the Klein-Gordon equation

$$\left[\bar{\square}_{\text{Lif}} - m^2 \right] \varphi = 0 \quad (10)$$

in the background of Lifshitz black hole spacetimes (6) which yields

$$(r^2 - r_+^2)\rho''(r) + \frac{5r^2 - 3r_+^2}{r}\rho'(r) + \frac{\ell^8\omega^2 - r^2(\ell^2k^2 + m^2\ell^2r^2)(r^2 - r_+^2)}{r^4(r^2 - r_+^2)}\rho(r) = 0 \quad (11)$$

for the ansatz $\varphi = \rho(r)e^{-i\omega t + ik\phi}$. Now we consider a tortoise coordinate r^* as

$$r^* = \frac{\ell^4}{r_+^3} \left[\frac{r_+}{r} - \text{arccoth} \left(\frac{r}{r_+} \right) \right], \quad (12)$$

which is defined by $dr^* = dr/f(r)$ with $f(r) = r^2(-M + r^2/\ell^2)/\ell^2$. Then $r \in [r_+, \infty]$ is mapped into $r^* \in [-\infty, 0]$. Introducing a new field $\Phi(= \sqrt{r}\rho(r))$ together with r^* , Eq.(11) can be written as the Schrödinger-type equation

$$\frac{d^2\Phi}{dr^{*2}} + \left[\omega^2 - V(r^*) \right] \Phi = 0, \quad (13)$$

where the potential $V(r)$ in r coordinate is given by

$$V(r) = \frac{7 + 4\ell^2 m^2}{4\ell^8} r^6 + \frac{4k^2 \ell^2 - 10\ell^2 M - 4\ell^4 m^2 M}{4\ell^8} r^4 - \frac{4k^2 \ell^4 M - 3\ell^4 M^2}{4\ell^8} r^2. \quad (14)$$

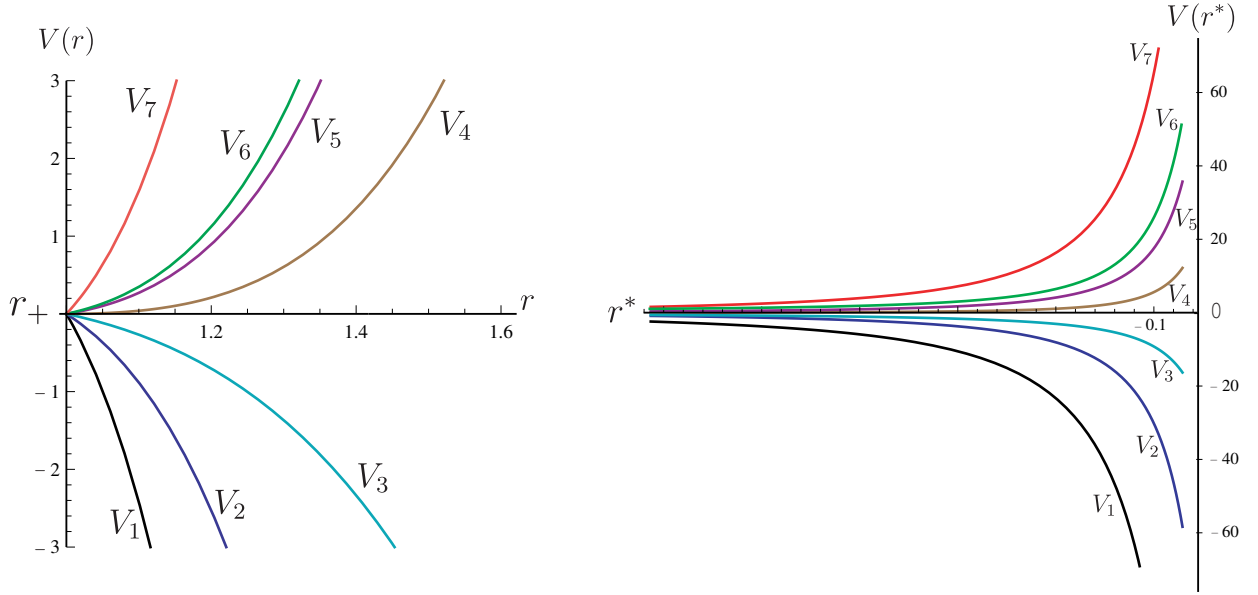


Figure 1: Potential V as a function of r (left panel) and r^* (right panel) with $M = 1$, $\ell = 1$, and $k = 0$. In these figures, $V_{1\sim 7}$ correspond to $m^2 \ell^2 = -9, -4, -2, -1, -1/2, 0, 2$, respectively. For $m^2 \ell^2 \geq -1$, the potentials ($V_{4\sim 7}$) are always positive for the whole range of $r_+ \leq r < \infty$ or $-\infty < r^* \leq 0$. However, the stability is extended to V_2 ($m^2 \ell^2 = -4$) because the scalar field is propagating in the Lifshitz spacetimes.

We could not obtain an analytic expression $V(r^*)$ written by r^* from (14) because it is difficult to express r in terms of r^* as (12) does show. However, $V(r^*)$ can be plotted in a parametric way by using (12) and (14). Fig. 1 shows that for $m^2 \ell^2 \geq -1$, the potential is always positive, which implies that the Lifshitz black hole is obviously stable under the scalar perturbation. Actually, there is correspondence between $V(r)$ and $V(r_*)$ for each m^2 [color matching], implying that the stability criterion remains unchanged. If one considers a scalar propagation in flat spacetimes, the stability condition is just the non-tachyonic mass of $m^2 \geq 0$. However, the stability condition was extended to the Breitenlohner-Freedman (BF) bound ($m^2 \geq m_{\text{BF}}^2 = -1/\ell^2$) in AdS_3 spacetimes [33], even if its potential is negative in AdS_3 spacetimes. Similarly, as was first mentioned in [3], the BF bound can be extended to the Lifshitz bound

$$m^2 \geq m_{\text{Lif}}^2 = -\frac{4}{\ell^4} \quad (15)$$

in Lifshitz spacetimes [28]. In other words, the Lifshitz black hole is unstable against the scalar propagation with mass ($m^2 < -4/\ell^2$) which is considered as the tachyonic instability.

In order to understand the Lifshitz bound (15) more explicitly, we investigate the potential

(14) at large r limit ($r_* \rightarrow 0$) which is approximated as an inverse square potential

$$V(r_*) \sim \frac{\xi}{r_*^2} \quad (16)$$

with

$$\xi = \frac{7 + 4m^2\ell^2}{36}. \quad (17)$$

This is because the first term in (14) dominates in this limit. Then, Eq. (13) becomes the Schrödinger equation with the inverse square potential. It is known that for this type of Schrödinger equation, the stability of the scalar field is determined by the condition of $\xi \geq -1/4$ [34] which yields the Lifshitz bound¹.

Finally, it is important to note that the Lifshitz bound corresponds to the stability condition obtained when applying the quasinormal mode approach. To see this explicitly, we consider the full form of QNFs [26, 27]

$$\begin{aligned} \omega = -i4\pi T_H \left[-1 - 2n - (4 + m^2\ell^2)^{1/2} \right. \\ \left. + \left(7 + \frac{3m^2\ell^2}{2} + \frac{k^2}{2M} + (3 + 6n)(4 + m^2\ell^2)^{1/2} + 6n(n+1) \right)^{1/2} \right] \end{aligned} \quad (18)$$

with $n = 0, 1, 2, \dots$. One checks that for s -mode ($k = 0$), the quasinormal frequency ω becomes negative imaginary if $m^2\ell^2$ satisfies the relation

$$\begin{aligned} \left[7 + \frac{3m^2\ell^2}{2} + (3 + 6n)(4 + m^2\ell^2)^{1/2} + 6n(n+1) \right]^{1/2} > 1 + 2n + (4 + m^2\ell^2)^{1/2} \\ \longrightarrow m^2\ell^2 \geq -4 \end{aligned} \quad (19)$$

which is exactly (15).

4 Absorption cross section

In order to find the absorption cross section, we introduce a new coordinate $x = (r^2 - r_+^2)/r^2$, which is useful to find a solution to Eq.(11). Then, the radial equation takes the form

$$\rho''(x) + \frac{1}{x(1-x)}\rho'(x) + \frac{\ell^2(\omega^2(1-x)^3 - m^2M^3x) - k^2M^2(1-x)}{4M^3(1-x)^2x^2}\rho(x) = 0. \quad (20)$$

¹Similarly, in the AdS₃ spacetimes [35], a permitted range for the massive scalar field with the inverse square potential is given by the BF bound: $\xi \geq -1/4 \rightarrow m^2 \geq m_{\text{BF}}^2 = -1/\ell^2$.

Here the prime (') denotes the differentiation with respect to the variable x . We note that $r \in [r_+, \infty)$ is mapped to a compact region of $x \in [0, 1)$. The solution to this equation is given by the confluent Heun (HeunC) functions as

$$\begin{aligned} \rho(x) = & C_1 x^\alpha (1-x)^\beta \text{HeunC}\left[0, 2\alpha, 2(\beta-1), \alpha^2, (\beta-1)^2 + \frac{1}{4M^3}(k^2 M^2 + \omega^2 \ell^2); x\right] \\ & + C_2 x^{-\alpha} (1-x)^\beta \text{HeunC}\left[0, -2\alpha, 2(\beta-1), \alpha^2, (\beta-1)^2 + \frac{1}{4M^3}(k^2 M^2 + \omega^2 \ell^2); x\right], \end{aligned} \quad (21)$$

where $C_{1,2}$ are arbitrary constants and

$$\alpha = i \frac{\omega \ell}{2M^{3/2}} = i \frac{\omega}{4\pi T_H}, \quad \beta = 1 + \sqrt{1 + \frac{m^2 \ell^2}{4}} \quad (22)$$

with T_H the Hawking temperature of Lifshitz black hole. In the neighborhood of the horizon ($x = 0$), using the property of $\text{HeunC}[0, a, b, c, d; 0] = 1$ leads to $\rho_0(x) = C_1 x^\alpha + C_2 x^{-\alpha}$ which yields

$$\begin{aligned} \varphi_0 = & C_1 e^{-i\omega t} x^\alpha + C_2 e^{-i\omega t} x^{-\alpha} \\ \sim & C_1 e^{-i\omega[t - \frac{1}{4\pi T_H} \ln x]} + C_2 e^{-i\omega[t + \frac{1}{4\pi T_H} \ln x]}. \end{aligned} \quad (23)$$

Here, the former corresponds to outgoing mode ($|_{x=0} \rightarrow$), while the latter to ingoing mode ($|_{x=0} \leftarrow$). Now we consider a scattering process where an ingoing mode comes from the spatial infinity and interacts with the black hole. Then, it is partially reflected backwards as outgoing mode to the infinity and the rest is absorbed into the black hole horizon. One way to achieve this goal is to introduce a purely ingoing wave at the horizon and then, carefully to decompose it into ingoing and outgoing waves in the large r region. The other approach is that we begin with ingoing and outgoing modes at the horizon and a purely outgoing mode at the spatial infinity. It is known that two pictures provide the same absorption cross section [36]. In this work, we use the former method to derive the absorption cross section of Lifshitz black hole.

To have the ingoing mode at the horizon, the constant C_1 is set to be zero in Eq.(23). We use the connection formula for the HeunC functions [27] to develop ingoing and outgoing modes at infinity ($x \rightarrow 1$). Then, Eq.(21) becomes

$$\rho_1(x) = C_2 \left((1-x)^\beta \frac{\Gamma(1-2\alpha)\Gamma(2-2\beta)}{\Gamma(3-2\beta+K)\Gamma(-2\alpha-K)} + (1-x)^{2-\beta} \frac{\Gamma(1-2\alpha)\Gamma(2\beta-2)}{\Gamma(2\beta-1+S)\Gamma(-2\alpha-S)} \right) \quad (24)$$

where K and S will be determined as the solutions to two algebraic equations

$$K^2 + (3 + 2\alpha - 2\beta)K + 2\alpha - 2\beta + 2 - \epsilon + \frac{\alpha^2}{2} = 0, \quad (25)$$

$$S^2 + (2\alpha + 2\beta - 1)S - 2\alpha + 4\alpha\beta - \epsilon + \frac{\alpha^2}{2} = 0 \quad (26)$$

with

$$\epsilon = \frac{1}{2}[1 - (1 - 2\alpha)(2\beta - 1)] - (\beta - 1)^2 - \frac{1}{4M^3}(k^2 M^2 + \omega^2 \ell^2). \quad (27)$$

Another way to find the solution at infinity is to start with the equation directly

$$\rho_\infty''(r) + \frac{5}{r}\rho_\infty'(r) + \frac{\ell^2}{r^2} \left(\frac{\ell^6 \omega^2}{r^6} - \frac{k^2}{r^2} - m^2 \right) \rho_\infty(r) = 0, \quad (28)$$

which was obtained by setting $r_+ = 0$ in Eq.(11). In order to find an analytic solution to (28), one has to focus on $k = 0$ (s -mode). For $k \neq 0$, it seems difficult to solve the equation. In this case, the s -mode solution is given in terms of Bessel functions

$$\rho_\infty^s(r) = \left(\frac{\ell^4 \omega}{2r^3} \right)^{\frac{2}{3}} \left[C_3 J_{-\gamma} \left(\frac{\ell^4 \omega}{3r^3} \right) \Gamma(1 - \gamma) + C_4 J_\gamma \left(\frac{\ell^4 \omega}{3r^3} \right) \Gamma(1 + \gamma) \right], \quad (29)$$

where $C_{3,4}$ are integration constants and γ is

$$\gamma = \frac{2}{3} \sqrt{1 + \frac{m^2 \ell^2}{4}} = \frac{2}{3}(\beta - 1). \quad (30)$$

For large r , the solution (29) can be written as

$$\rho_\infty^s(r) = \tilde{C}_3 \left(\frac{1}{r^2} \right)^{2-\beta} + \tilde{C}_4 \left(\frac{1}{r^2} \right)^\beta \quad (31)$$

with $\tilde{C}_{3,4}$ constants.

It is worth noting that to have a regular behavior (normalizable solution) at $r \rightarrow \infty$ [38], β should be confined to $1 \leq \beta \leq 2$ which corresponds to

$$-\frac{4}{\ell^2} = m_{\text{Lif}}^2 \leq m^2 \leq 0. \quad (32)$$

Imposing the regularity at infinity restricts the stability condition (15) to a smaller region (32). According to the AdS₃-analysis [39], it is shown in the context of AdS/CFT correspondence that if the mass of a bulk scalar lies in the interval

$$-\frac{1}{\ell^2} = m_{\text{BF}}^2 \leq m^2 \leq 0, \quad (33)$$

a single gravity theory in the bulk describes two different conformal field theories on the boundary. Similarly, we might confine the allowed mass range to (32) in Lifshitz spacetimes. In other words, (32) is necessary to compute the absorption cross section of a massive scalar field propagating in the Lifshitz spacetimes.

We are now in a position to obtain the absorption cross section. For this purpose, we have to decompose $\tilde{C}_{3,4}$ into the ingoing and outgoing coefficients. However, distinguishing

between ingoing and outgoing modes at asymptotic region is not a trivial task because the spacetime is Lifshitz. Following the ansatz in asymptotically AdS spacetimes, we introduce new constant parameters C_{in} and C_{out} given by [37, 38]

$$\tilde{C}_3 = C_2(C_{\text{in}} + C_{\text{out}}), \quad \tilde{C}_4 = icC_2(C_{\text{out}} - C_{\text{in}}), \quad (34)$$

where c is a parameter with the length dimension $[L]^{4\beta-4}$. For the ansatz (34), comparing (31) with (24) leads to

$$C_{\text{in}} = \frac{\Gamma_2}{2} + i\frac{\Gamma_1}{2c}, \quad C_{\text{out}} = \frac{\Gamma_2}{2} - i\frac{\Gamma_1}{2c}, \quad (35)$$

where $\Gamma_{1,2}$ are given by

$$\Gamma_1 = r_+^{2\beta} \frac{\Gamma(1-2\alpha)\Gamma(2-2\beta)}{\Gamma(3-2\beta+K)\Gamma(-2\alpha-K)}, \quad (36)$$

$$\Gamma_2 = r_+^{4-2\beta} \frac{\Gamma(1-2\alpha)\Gamma(2\beta-2)}{\Gamma(2\beta-1+S)\Gamma(-2\alpha-S)}. \quad (37)$$

It is well-known that the conserved flux $\mathcal{F}(r)$ is defined by

$$\mathcal{F}(r) = \frac{\sqrt{-g}g^{rr}}{2i} (\rho^* \partial_r \rho - \rho \partial_r \rho^*). \quad (38)$$

Using this expression, the absorption coefficient (\mathcal{A}) is given by

$$\mathcal{A} = \left| \frac{\mathcal{F}_{r_+}^{\text{in}}}{\mathcal{F}_{\infty}^{\text{in}}} \right|, \quad (39)$$

where \mathcal{F}_{r_+} and \mathcal{F}_{∞} denote the flux at the horizon and at asymptotic region ($r \rightarrow \infty$), respectively. These are computed as

$$\mathcal{F}_{r_+}^{\text{in}} = -\omega r_+ |C_2|^2, \quad \mathcal{F}_{\infty} = \frac{4c(\beta-1)|C_2|^2}{\ell^4} \left[|C_{\text{in}}|^2 - |C_{\text{out}}|^2 \right]. \quad (40)$$

Then, we have \mathcal{A} as

$$\mathcal{A} = \frac{\omega \ell^4 r_+}{4c(\beta-1)|C_{\text{in}}|^2}. \quad (41)$$

Finally, the absorption cross section σ_{abs} takes the form

$$\sigma_{\text{abs}} = \frac{\mathcal{A}}{\omega} = \frac{\ell^4 r_+}{4c(\beta-1)|C_{\text{in}}|^2}, \quad (42)$$

which looks like an unclear form.

Hence, we consider the absorption cross section in the low energy and massless limits. To investigate the absorption cross section in the limits of $\omega \rightarrow 0$ and $m \rightarrow 0$ (equivalently, $\alpha \rightarrow 0$ and $\beta \rightarrow 2$), we rewrite Eq.(42) as

$$\sigma_{\text{abs}} = \frac{\ell^4 r_+}{c(\beta - 1)|\Gamma_2 + i\Gamma_1/c|^2}, \quad (43)$$

where $\Gamma_{1,2}$ are given by (36) and (37). We mention that in the limits of $\omega \rightarrow 0$ and $m \rightarrow 0$, the parameters K and S are determined as

$$K = 0 \text{ or } 1, \quad S = -1 \text{ or } -2, \quad (44)$$

by solving (25) and (26). There are four combinations of $(K, S) = (0, -1), (0, -2), (1, -1)$ and $(1, -2)$ which show a feature of the Lifshitz black hole. It turns out that four combinations provide a single combination for Γ_1 and Γ_2 as

$$\Gamma_1 = 0, \quad \Gamma_2 = 1. \quad (45)$$

Substituting this into (43) leads to the absorption cross section

$$\sigma_{\text{abs}}^{\{\omega \rightarrow 0, m \rightarrow 0\}} = 2\pi r_+ = A, \quad (46)$$

where we have chosen $c = \frac{\ell^4}{2\pi}$ for recovering the horizon area. We have to say that there is no information to fix c in the wave equation approach.

Consequently, it is shown that the absorption cross section for scattering of the scalar off the $z = 3$ Lifshitz black hole is given by the area of the horizon in the limits of $\omega \rightarrow 0$ and $m \rightarrow 0$.

5 Discussions

First of all, we would like to mention the stability of Lifshitz black hole in three dimensions. Even though the potential is positive definite for $m^2 \ell^2 \geq -1$, the stability condition is given by the Lifshitz bound (15) when using a minimally coupled scalar propagating in the Lifshitz black hole spacetimes. This stability condition was confirmed by the quasinormal mode approach to the Lifshitz black hole.

For the mass range (32), we have computed the absorption cross section by considering scattering of a minimally coupled scalar off the Lifshitz black hole. The absorption cross section reduces to the horizon area in the low energy and massless limits of s -wave propagation, indicating that the Lifshitz black hole also satisfies the universality of low-energy absorption cross section for black holes.

Finally, we propose that the decaying rate is given by [37, 40]

$$\Gamma_{\text{Lif}} = \frac{\sigma_{\text{abs}}}{e^{\frac{\omega}{T_H}} - 1}, \quad (47)$$

where σ_{abs} is given by (43). This decaying rate could be calculated by the boundary field theory if the latter is known [41]. However, we do not know exactly the boundary field theory which may exist by presuming the Lif/CFT correspondence. We expect to recover the proposed decaying rate (47) from the boundary field theory approach in the near future.

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